



Fig. 1 Test of the constancy of the eddy viscosity in the ratio portion of a turbulent boundary layer over a flat plate with uniform blowing. (Data taken for air-air system.<sup>3</sup>)

outer region is independent of pressure gradient or perturbing influence at the wall. It was pointed out earlier by this author<sup>3</sup> that if this was indeed the situation, as in the case of a pressure gradient, the eddy diffusivity of the outer region could also be treated as a constant with mass addition at the wall, provided the momentum of injection normal to the plate did not exceed the momentum of the shear layer. Thus, using the Clauser expression for the eddy viscosity

$$\epsilon = k \cdot \bar{U}_\infty \cdot \delta^* \rho_\infty \quad (2)$$

where  $k$  is a function of  $\delta^*/\tau_0$  and the injection parameter  $V_0\rho_0/U_\infty\rho_\infty$ , one can investigate various equilibrium velocity profiles with uniform wall injection to determine the behavior of  $\epsilon$  in the outer portion of the layer.

The results of such a calculation for air injected uniformly along the plate and normal to the main air flow is shown in Fig. 1 for an assumed constant  $k = 0.018$ . It is clear that up to the so-called "blow-off" point, the assumptions maintained by Clauser for eddy viscosity of the Reynolds stress layer are still correct, and it appears that we are justified in considering  $k$  as a constant in calculating  $\epsilon$  in the region before separation. The variation of  $\epsilon$  may be attributed at this stage to experimental inaccuracies in the experimental work.

The main problem appears to be centered on the nature of the wall shear layer under the influence of mass addition. The fact is that only for injection ratios of less than the separation value can the profile be described by the Clauser model. As yet there is no process method by which the shear velocity  $U_\tau^*$  can be related to the mass addition parameter,  $V_0\rho_0/U_\infty\rho_\infty$ . However, it appears from the data of Fig. 1 and Ref. 3, provided the mass addition ratio is less than 0.01, that the outer shear layer is unaffected by conditions at the wall. The observation by the authors is essentially correct, but one must wait upon results of additional measurements made close to the wall for a broader range of injection parameters before accepting this model.

## References

- 1 Mickley, H. S. and Smith, K. A., "Velocity defect law for a transpired turbulent boundary layer," *AIAA J.* 1, 1685 (1963).
- 2 Clauser, F. H., "The turbulent boundary layer," *Advances in Applied Mechanics* (Academic Press, Inc., New York, 1956), Vol. 4, pp. 1-51.
- 3 Hacker, D. S., "Interferometric investigation of the stability of a turbulent boundary layer with mass addition," *Am. Soc. Mech. Engrs. Paper 58A-249* (1958).

## Reply by Authors to D. S. Hacker

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THE authors appreciate the interest of Hacker in their earlier paper.<sup>1</sup> The purpose of that paper was to present evidence that when the friction velocity is based upon the maximum shear, the outer portion of a transpired turbulent boundary layer may be described by a velocity defect law similar to that which describes nontranspired boundary layers. Additional evidence is certainly desirable and the authors are currently carrying out additional experimental measurements.

The maximum shear appears to provide a more suitable boundary condition for the outer flow than the wall shear. If, as the experimental data indicate, the eddy viscosity of the outer flow is independent of  $y$ , an analysis along the lines of Clauser<sup>2</sup> as suggested by Hacker<sup>3</sup> is possible. The authors have embarked on such calculations.

The problem of treating the inner flow and of connecting it with the outer flow remains to be solved.

## References

- 1 Mickley, H. S. and Smith, K. A., "Velocity defect law for a transpired turbulent boundary layer," *AIAA J.* 1, 1685 (1963).
- 2 Clauser, F. H., "The turbulent boundary layer," *Advances in Applied Mechanics* (Academic Press, Inc., New York, 1956), Vol. 4, pp. 1-51.
- 3 Hacker, D. S., "Interferometric investigation of the stability of a turbulent boundary layer with mass addition," *Am. Soc. Mech. Engrs. Paper 58-A-249* (1958).

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